

A fractional order model for lead-acid battery crankability estimation

J. Sabatier^{a,*}, M. Cugnet^a, S. Laruelle^b, S. Grugeon^b, B. Sahut^c, A. Oustaloup^a, J.M. Tarascon^b

^aIMS – LAPS, UMR 5218 CNRS, Bât. A4, 351 cours de la Libération, 33405 Talence Cedex, France

^bLRCS, UMR 6007 CNRS, 33 rue Saint Leu, 80039 Amiens Cedex, France

^cPSA Peugeot Citroën – CTV – VV187, 2, route de Gisy, 78943 Velizy-Villacoublay Cedex, France

ARTICLE INFO

Article history:

Received 25 April 2009

Received in revised form 30 May 2009

Accepted 30 May 2009

Available online 7 June 2009

PACS:

02.30.Yy Control theory

82.47.Cb Lead-acid battery

89.40.Bb Land transportation

Keywords:

Fractional systems

System identification

Lead-acid batteries

Vehicle cranking

ABSTRACT

With EV and HEV developments, battery monitoring systems have to meet the new requirements of car industry. This paper deals with one of them, the battery ability to start a vehicle, also called battery crankability. A fractional order model obtained by system identification is used to estimate the crankability of lead-acid batteries. Fractional order modelling permits an accurate simulation of the battery electrical behaviour with a low number of parameters. It is demonstrated that battery available power is correlated to the battery crankability and its resistance. Moreover, the high-frequency gain of the fractional model can be used to evaluate the battery resistance. Then, a battery crankability estimator using the battery resistance is proposed. Finally, this technique is validated with various battery experimental data measured on test rigs and vehicles.

© 2009 Elsevier B.V. All rights reserved.

1. Introduction

More and more drastic automobile pollution norms and oil price increase lead car manufacturers to design new vehicles (stop and start, hybrid, or electric). The underlying idea is to reduce exhaust emissions in the built-up areas, either by stopping motor when the vehicle is not moving or by a substitution of fossil fuel by electric fuel.

To keep these new vehicles in good working order, car manufacturers must integrate a reliable electrical energy storage management in their vehicles. For electrical energy storage, various batteries, fuel cells and supercapacitors are actually studied. For the electrical energy management, State Of Charge (SOC), State Of Health (SOH) and also State Of Function (SOF) estimators of the storage system must be designed.

Many reliable battery state estimators were proposed all over the world. Most of them deal with SOC estimation exclusively [1,2]. The interested reader can consult [3] for a presentation of some methods and [4] for a specific application to battery monitoring in future vehicles. Nevertheless, a reliable battery monitoring system design remains a difficult task because on one hand, the estimator must have all the following advantages:

- good estimation accuracy,
- real-time working,

* Corresponding author. Tel.: +33 540 006 607.

E-mail addresses: jocelyn.sabatier@laps.ims-bordeaux.fr, jocelyn.sabatier@u-bordeaux1.fr (J. Sabatier).

URLs: <http://www.u-picardie.fr/labo/lrsc/> (S. Laruelle), <http://www.psa-peugeot-citroen.com/en/hp1.php> (B. Sahut).

- immediate estimation,
- robustness to operating conditions.

On the other hand, a battery is a complex electrochemical system whose behaviour is:

- non-linear in relation to a current input signal,
- sensitive to temperature and age,
- with long memory due to diffusion phenomena.

In this paper, only lead acid batteries are studied because they are the most widely used in cars. However, the SOF estimator proposed could be easily extended to other battery types. This method is based on system identification by a fractional order model of the battery behaviour. Fractional behaviour of lead-acid batteries has been demonstrated for a long time by chemists but only few existing estimation methods take into account this behaviour. Such a situation can surely be explained by the lack of tools for fractional systems analysis and modelling until fifteen years ago. These tools now exist [5–7]. In system identification field, several methods were created [8,9] and permit now to take into account systems that exhibit long memory phenomena (diffusion processes). Interest of these methods is the computation of low order (and thus low parameter number) models. This property is used in this paper for the lead-acid batteries behaviour modelling. Given the small number of parameters in the obtained model, a simple and efficient estimation method can be designed. The proposed method is particularly attractive for SOF estimation when the vehicle starts. It takes into account operating temperature changes and the long relaxation time of the battery. The high current level (about 1 kA), supplied by the battery to the starter, is especially used to provide information on the battery power availability and consequently the battery crankability.

Fractional differentiation has found numerous applications in fields like physics [20], finance [24], chemistry [23], bioengineering [21]. This paper is thus a new application in engineering [22] and is organized as follow. Section 2 shows that a vehicle cranking is an interesting operating phase to evaluate the battery resistance or its lack of power. Section 3 highlights the fractional order behaviour of a lead-acid battery and presents the fractional order model used to characterize this behaviour. System identification method implemented in the SOF estimation method is described in Section 4. The identification method is then applied to signals recorded during a vehicle cranking. The battery resistance and crankability estimation method are detailed in Section 5.

2. Interest in vehicle cranking signals

It is no longer possible to provide battery monitoring systems without taking into account the ageing. In practice, such systems would not be reliable during all the battery service life. In electrochemistry, the battery ageing can be defined as an irreversible degradation process of its performances due to various phenomena (sulphating, corrosion, loss of active material ...) resulting in the battery use. In the automobile industry, the battery ageing can also be defined as a gradual and irreversible incapacity to supply energy and power required for the vehicle functions.

The battery breakdowns mostly occurred during the coldest months of the year, because the battery is less powerful at low temperature. If the vehicle does not start and if it is really due to the battery, two situations can take place:

- 1 – the battery is discharged, then a recharge is enough,
- 2 – the battery is dead, a replacement is required.

Battery breakdowns are expensive not only for the customers but also for the car manufacturers during the warranty period. As a result, a reliable battery state estimator is important for both customers and car manufacturers to discriminate, for instance, the previous situations 1 and 2.

A good way to achieve this goal is to analyze in details the cranking function, because this is the most important function in the vehicle. The notion of crankability can be defined as the battery capability to start the vehicle engine.

Current and voltage signals of Fig. 1 have been recorded on a cranking vehicle. The duration of each voltage and current signals arcs is equal to half an engine revolution. It is a consequence of the four-stroke cycle (during two revolutions of the crankshaft) operating in car combustion engines.

The cranking produces a low discharge of the battery. The capacity of the battery used to produce signals in Fig. 1 is equal to 60Ah or 216,000C (1Ah = 3600C), which greatly exceeds the 300C required to start the engine (example in Fig. 1). On the other hand, the cranking needs a very high power, exceeding several kilowatts (Fig. 2). The vehicle cranking thus provides interesting information on the battery available power without modifying its state.

Also, the resistance of a new battery is approximately equal to 5 m Ω . Thus, a 9 A pulse will produce (about) a 45 mV battery voltage variation, compared to 4.5 V for a 900 A pulse. Obviously, the higher the current, the easier and more accurate the resistance measurement (better signal-to-noise ratio).

Moreover, a battery resistance estimation method based on cranking is not expensive, because there is no need to apply a special signal to the battery (generated by an additional device). Such an analysis permits to conclude that the cranking is a really interesting vehicle operating phase to estimate the battery resistance.

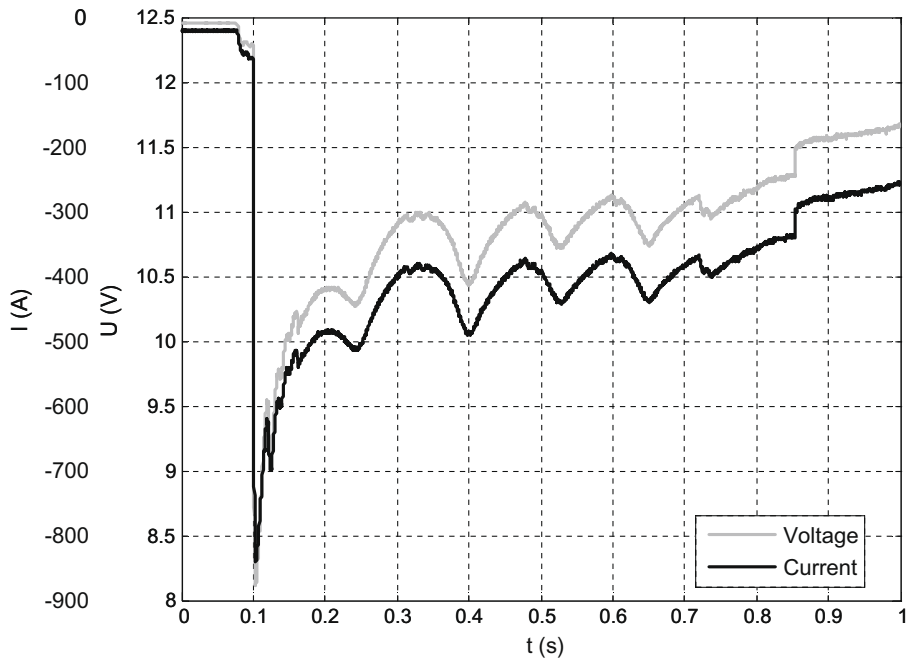


Fig. 1. Example of cranking current and voltage measured with a new battery.

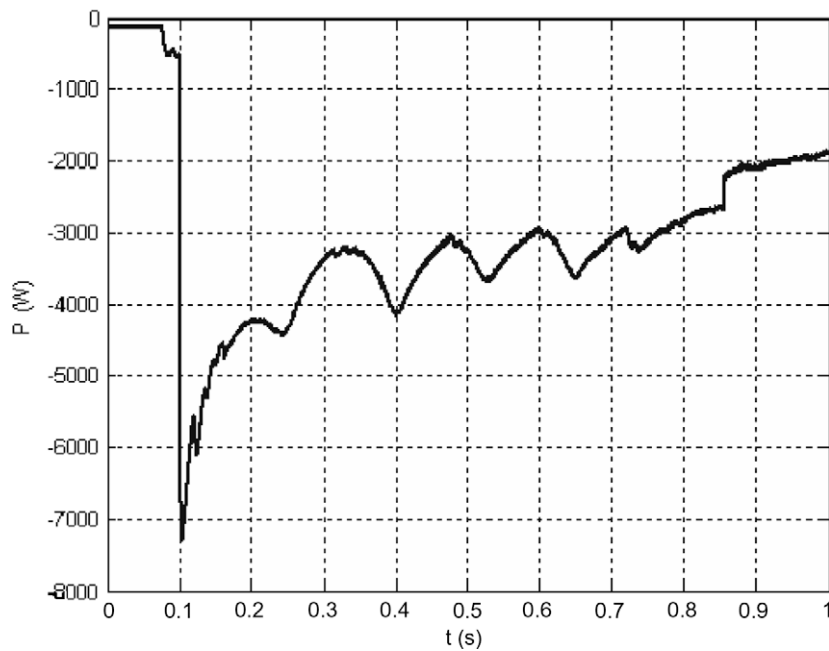


Fig. 2. Power supplied by a new battery during vehicle cranking.

Fig. 3 shows the impact of a genuine vehicle starting on the battery voltage at various SOC. The battery voltage seems to be clearly dependent on the SOC. The cranking can be viewed not only as the main vehicle function, but also as a signal generation that can be used to estimate the SOC. Fig. 3 analysis reveals two interesting details:

- the Open Circuit Voltage (OCV) linear evolution according to the SOC at $t = 0$,
- the resistance is constancy for all SOC higher than 50%.

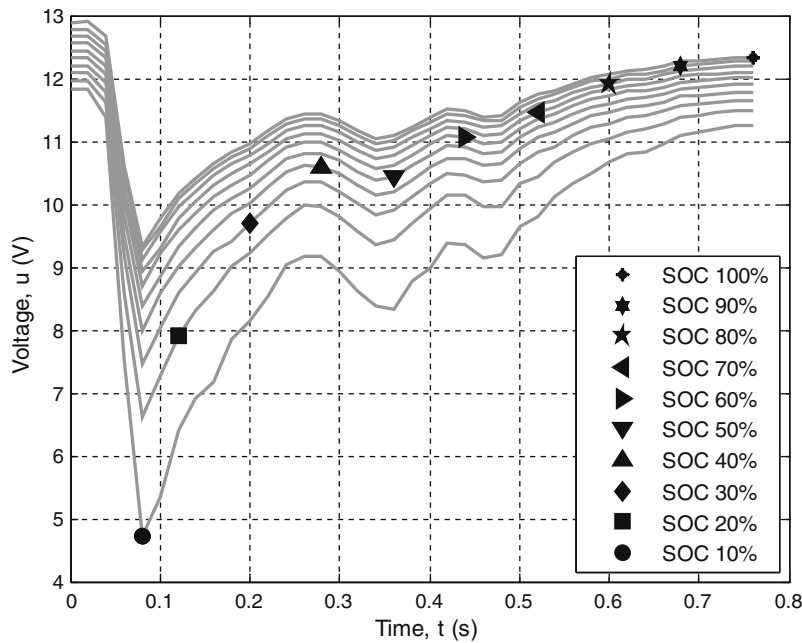


Fig. 3. Voltage of a new battery at various SOC with the same cranking current.

The main drawback of the cranking is its non-repeatability. Each cranking is unique because the cranking current depends not only on the starter state, but also on the engine state (crankshaft position, temperature, wear and tear).

The non-repeatability of the vehicle cranking makes impossible an approach simply based on the voltage signal to detect the battery ageing or a lack of power. A model-based approach is thus preferable as shown in the next sections in which a fractional order model is used. This fractional order model is now defined.

3. A fractional model of lead-acid batteries for cranking signals

It is possible to demonstrate the link existing between a fractional system and a system described by a diffusive equation (parabolic differential equation). This demonstration can be done using the work presented in [10]. For the same reasons, the Fick's Laws that govern the ionic diffusion in the battery justify the introduction of fractional systems to model the electrical battery behaviour.

The Randles model is frequently used in the lead-acid battery modelling literature. This model results from the simplified resolution of the electrochemical diffusion equations in batteries [11]. If $u(t)$ denotes voltage variations in relation to the open circuit voltage and if $i(t)$ denotes the battery current, Randles model is defined by the electric circuit of Fig. 4.

The Randles model fractional behaviour is due to the fractional impedance $W(s)$. This impedance is a fractional order integrator of order $n = 0.5$, called the Warburg cell. In the Laplace's domain, the Warburg cell is indeed characterized by the transmittance [12]:

$$\forall \sigma \in \mathbb{R} \quad W(s) = \frac{\sigma}{s^n}. \quad (1)$$

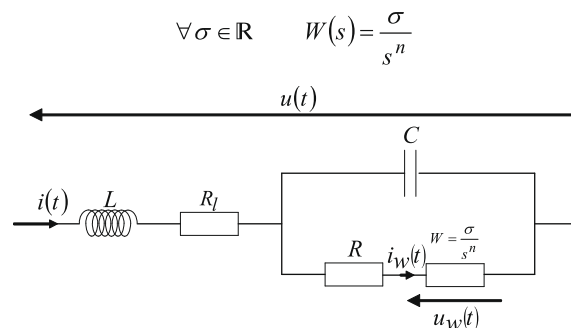


Fig. 4. Randles' model of a lead-acid battery.

Fig. 4 Randles model transfer function is thus given by:

$$H(s) = \frac{U(s)}{I(s)} = (Ls + R_l) + \frac{\frac{1}{Cs} (R + \frac{\sigma}{s^n})}{(\frac{1}{Cs} + R + \frac{\sigma}{s^n})}, \quad (2)$$

which can be simplified if $L = 0$:

$$H(s) \underset{L=0}{=} \frac{R_l R C s^{n+1} + R_l \sigma C s + (R_l + R) s^n + \sigma}{s^n (R C s + \sigma C s^{1-n} + 1)}. \quad (3)$$

Relation (3) thus demonstrates that the Randles model frequently used for the battery modelling is a fractional model [13,14]. However, if the battery study is limited to the frequency range [8 Hz; 30 kHz] corresponding to the vehicle cranking signals spectrum, a simpler model can be used to describe the dynamic behaviour of a lead acid battery. This simpler model is defined by:

$$H_{CRONE}(s) = K \left(\frac{1 + \frac{s}{\omega_h}}{1 + \frac{s}{\omega_b}} \right)^n. \quad (4)$$

The previous model is also a fractional model, but it is composed of four parameters instead of six for the Randles model. The next section describes how the numeric values of these parameters are computed through system identification.

4. Fractional model identification

The parametric estimation of $H_{CRONE}(s)$ model given by (4) consists in a computation of the parameter vector:

$$\theta = [K \quad \omega_b \quad \omega_h \quad n]^T. \quad (5)$$

It is based on an output error approach (Fig. 5), previously applied to the modelling of thermal systems [15].

4.1. Output error identification algorithm

The sampled data set is composed of K data pairs $\{i(kh), u^*(kh)\}$ (h being the sampling period) where:

$$u^*(kh) = u(kh) + b(kh), \quad (6)$$

b being an output white noise.

Vector θ being an estimation of θ , the output prediction error is given by:

$$\varepsilon(kh, \hat{\theta}) = u^*(kh) - \hat{u}(kh, \hat{\theta}). \quad (7)$$

The optimal value of $\hat{\theta}$, (θ_{opt}) is then obtained by the quadratic criterion minimization:

$$J(\hat{\theta}) = \sum_{k=0}^{K-1} \varepsilon^2(kh, \hat{\theta}). \quad (8)$$

The output prediction $\hat{u}(kh, \hat{\theta})$ being non-linear in $\hat{\theta}$, the Marquardt algorithm [16], a non-linear programming technique, is used to estimate $\hat{\theta}$ iteratively:

$$\theta_{i+1} = \theta_i - \left\{ [J''_{\theta\theta} + \xi I]^{-1} J'_{\theta} \right\}_{\theta=\theta_i}, \quad (9)$$

$$\text{with } \begin{cases} J'_{\theta} = -2 \sum_{k=0}^{K-1} \varepsilon(kh) S(kh, \hat{\theta}) : \text{gradient} \\ J''_{\theta\theta} \approx 2 \sum_{k=0}^{K-1} S(kh, \hat{\theta}) S^T(kh, \hat{\theta}) : \text{hessian} \\ S(kh, \hat{\theta}) = \frac{\partial \hat{u}(kh, \hat{\theta})}{\partial \theta} : \text{output sensitivity function} \\ \xi : \text{monitoring parameter} \end{cases}$$

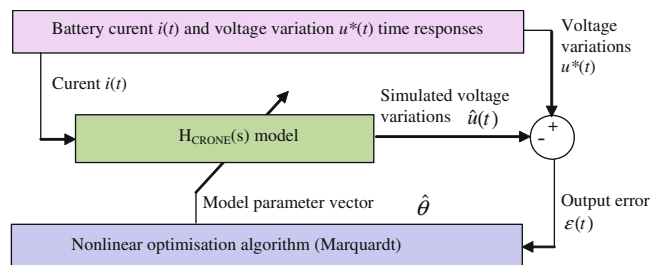


Fig. 5. Parametric estimation of $H_{CRONE}(s)$ models by output error approach.

This algorithm, often used in non-linear optimization, ensures robust convergence, even if $\hat{\theta}$ is initialized at a value far from the optimal value [17,18].

4.2. Output sensitivity computation

The output sensitivity of each $H_{CRONE}(s)$ model parameter is computed using partial differentiation of the output prediction:

$$\frac{\partial \hat{u}(t, \hat{\theta})}{\partial \hat{\theta}_i} = \mathcal{L}^{-1} \left(\left[\frac{\partial H_{CRONE}(s)}{\partial \theta_i} \right]_{\theta_i = \hat{\theta}_i} \right) * i(t), \quad (10)$$

$$\text{where : } \frac{\partial H_{CRONE}(s)}{\partial K} = \frac{1}{K} H_{CRONE}(s), \quad (11)$$

$$\frac{\partial H_{CRONE}(s)}{\partial \omega_b} = \frac{ns}{\omega_b^n} \left(1 + \frac{s}{\omega_b} \right)^{-1} H_{CRONE}(s), \quad (12)$$

$$\frac{\partial H_{CRONE}(s)}{\partial \omega_h} = -\frac{ns}{\omega_h^n} \left(1 + \frac{s}{\omega_h} \right)^{-1} H_{CRONE}(s), \quad (13)$$

$$\frac{\partial H_{CRONE}(s)}{\partial n} = \ln \left(\frac{1 + \frac{s}{\omega_b}}{1 + \frac{s}{\omega_h}} \right) H_{CRONE}(s). \quad (14)$$

Relations (11)–(13) can be simulated using the integer derivative approximation [5] and relation (14) and also using the appendix from [19].

4.3. Validation of the estimated parameters

Assuming that the prediction error is a zero mean white sequence, the estimated parameter covariance matrix is given by [17,18]:

$$\text{cov}(\hat{\theta}) = \sigma^2 \left(\sum_{k=0}^{K-1} S(kh, \hat{\theta}) S^T(kh, \hat{\theta}) \right)^{-1}. \quad (15)$$

Using this matrix, the parameter variances (on the matrix diagonal) and the correlation coefficients between parameters (not on the matrix diagonal) can be obtained.

4.4. Application to cranking signals

Model (4) and the identification method previously described have been applied to voltage and current signal measured on battery terminal during a cranking. The current signal corresponds to the input signal. Fig. 6 shows comparisons of the voltage signal with the signal produced by the obtained model. This comparison validates the model and the identification method.

Nevertheless, it would be interesting to check if such a result can be obtained only with an equivalent (in terms of parameter number) integer model. Such a model, with five parameters, is defined by:

$$H_{INTEGER}(s) = K \frac{\left(1 + \frac{s}{\omega_{h1}} \right) \left(1 + \frac{s}{\omega_{h2}} \right)}{\left(1 + \frac{s}{\omega_{b1}} \right) \left(1 + \frac{s}{\omega_{b2}} \right)}. \quad (16)$$

In Fig. 6a and b, a comparison of the results obtained with models (4) and (16) is proposed. One can note that the mean relative error obtained with the integer and the fractional models are respectively equal to 0.99% and 0.33%. Both models give a good simulation of the battery voltage behaviour in cranking, but the fractional model gives a result three times better than the integer model, with only four parameters (versus 5 for the integer model). For cranking modelling, a fractional order model really improves the simplicity/accuracy compromise.

5. Crankability estimator

The vehicle cranking depends on the available battery power, which is the product of the current with the voltage measured on the battery terminals. The cranking function is well ensured if:

- the current, proportional to the starter torque, is higher than the stationary engine resistant torque,
- the voltage, proportional to the starter speed, is higher than the reset threshold of the engine micro controller.

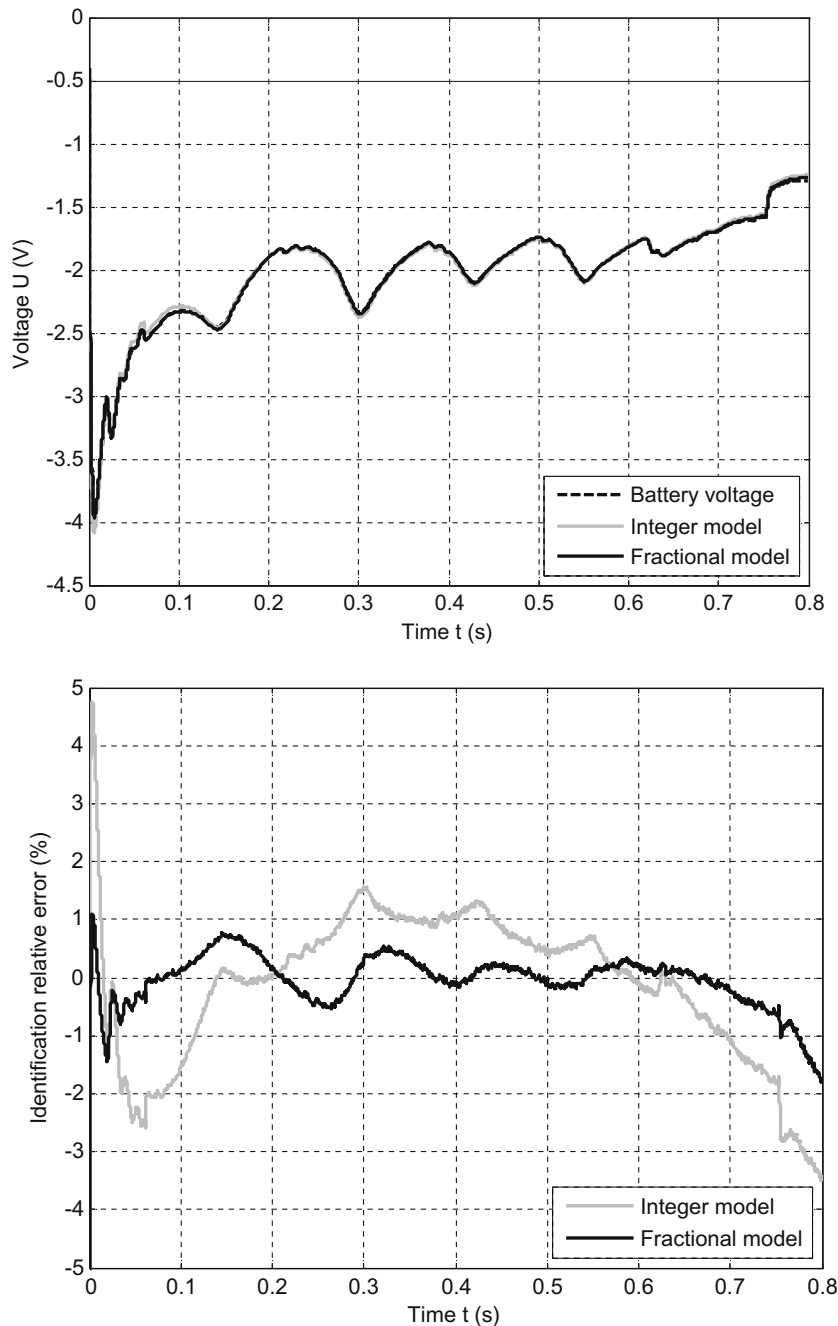


Fig. 6. Identification of both models with vehicle cranking data.

The SOF_C is a State Of Function equals to the ratio of the available capacity with the rated capacity. Some vehicle cranking tests carried out on three new batteries (NJS2, NJS1, NJS0) and three used batteries (VCN101, VCN102, VCN103) have shown the impact of the SOF_C on the battery power performance, as presented in Fig. 7.

An analysis of Fig. 7 reveals that the vehicle does not start when $\text{SOF}_C < 10\%$, except for VCN101. The reason for the non-cranking is not the lack of energy but a lack of power. So, a threshold of power equals to about 4 kW (at 20 °C) seems to be an appropriate indicator of the battery crankability. However, the reliability of this indicator is conditioned by the threshold temperature sensitivity. The CRONE model (4) used to simulate the battery cranking behaviour permits to overcome this problem. With such a fractional order model, the battery resistance estimation and the available power estimation can be carried out.

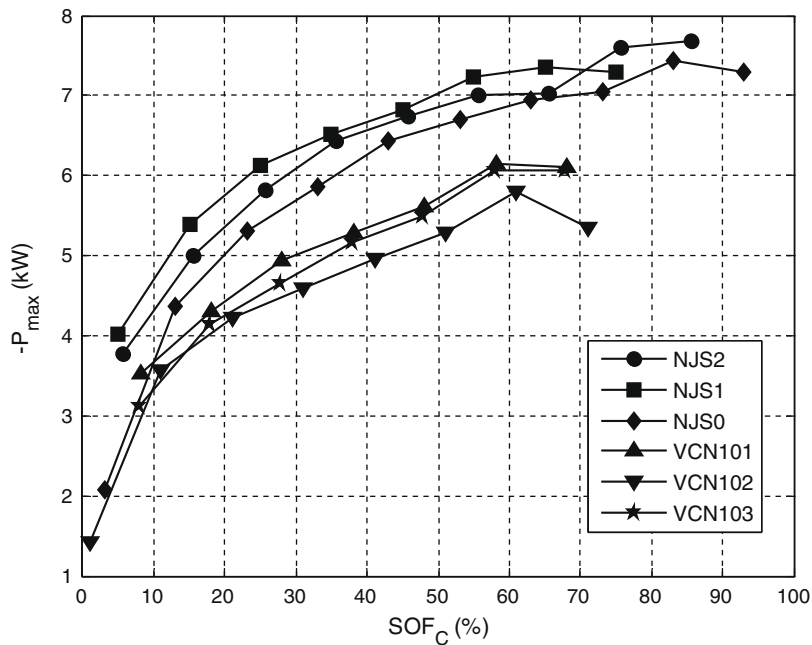


Fig. 7. Comparison of the battery maximal power provided by the battery during cranking for various SOF_C at 20 °C.

Frequency responses of several CRONE models, obtained from data measured with an used battery at different SOC, are given in Fig. 8. On these curves, one can note that the battery resistance is equal to the CRONE model high-frequency gain (imaginary part equals to 0). It is thus possible to estimate the battery resistance from a $H_{\text{CRONE}}(s)$ model identification during cranking. Such an estimation is accurate because with the data measured on vehicle, the identification errors of the $H_{\text{CRONE}}(s)$ models are always below 5%.

On Fig. 8, one can also notice that the high-frequency gain, and thus the resistance, is directly linked to the SOF_C . The higher is the resistance and the lower is the SOF_C . A $H_{\text{CRONE}}(s)$ model identification during cranking thus permits to evaluate the battery SOF_C and, according to the comments done in Fig. 7, the available power.

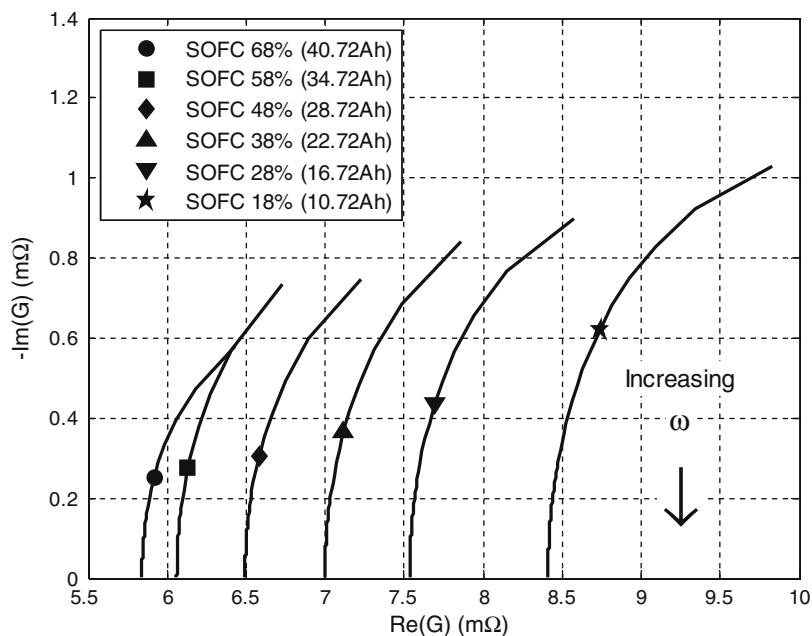


Fig. 8. Nyquist Diagrams of the VCN101 battery identified models at 20 °C.

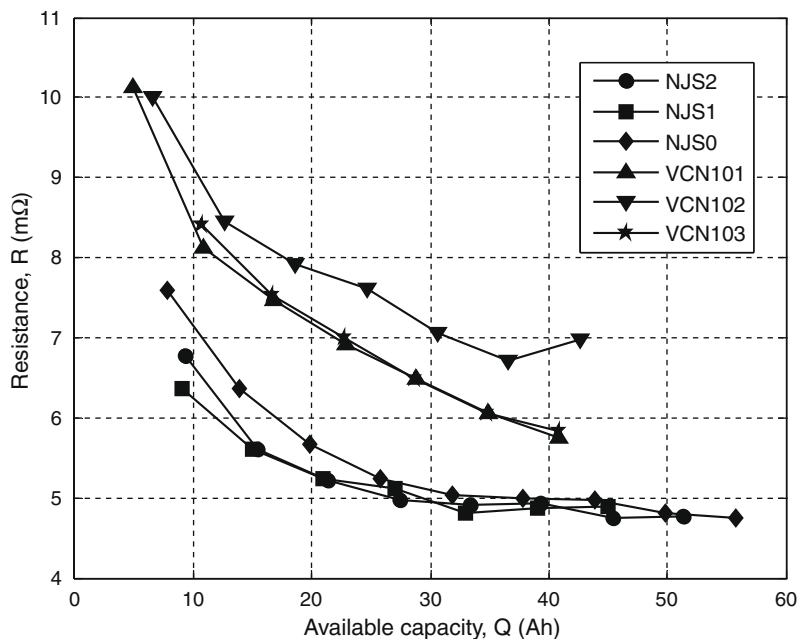


Fig. 9. Battery resistance variations with the stored energy for new and aged batteries.

Fig. 9 represents the battery resistances as a function of the stored energy for three new batteries (NJS2, NJS1, NJS0) and three used batteries (VCN101, VCN102, VCN103). This figure proves that the battery resistance depends on the available capacity, but also that the resistance of a new battery is really different from the resistance of an used battery for a given capacity. Combined with the capacity estimator, a battery resistance estimator is thus a really interesting age estimator.

Note that the battery resistance increases when:

- the temperature decreases,
- the state of charge decreases,
- the state of health decreases.

6. Conclusion

This work has shown that the electrical signals measured on a battery terminals during cranking are really interesting signals to design a battery resistance estimator or a crankability estimator. This paper has also shown that the battery crankability only depends on the available power. This power can be estimated from the battery resistance. A good way to measure the battery resistance is to evaluate the gain in high frequency of a simple fractional model (only 4 parameters) identified on the electrical signals recorded during cranking. The identification method is detailed in the paper and its efficiency is demonstrated. However it cannot be used for a real-time implementation in a car microcontroller. Thus, the authors will propose in another paper a way to estimate the resistance in order to provide a suitable solution for the car industry. This work is a new application of fractional differentiation in engineering and show the interest of fractional modelling for battery monitoring. Given the promising results obtained on lead acid batteries we intend now to extend the proposed tool to others batteries but also for other electrochemical systems.

Acknowledgement

The authors thank PSA Peugeot Citroën for support of this work and for permission to publish this article.

References

- [1] Pang S, Farrell J, Du J, Barth M. Battery state-of-charge estimation. in: Proceedings of the American control conference. Arlington, VA; 2001. p. 1644–9.
- [2] Piller S, Perrin M, Jossen A. Methods for state of charge determination and their applications. J Power Sour 2001;96:113–20.
- [3] McAndrews JM, Jones RH. A Valve regulated lead-acid battery Management System (VMS). In: Proceedings of the 18th IEEE INTELEC, 1996; p. 507–13.
- [4] Meissner E, Richter G. Battery monitoring and electrical energy management precondition for future vehicle electric power systems. J Power Sour 2003;116:79–98.
- [5] Oustaloup A. La dérivation non entière, théorie, synthèse et applications. In: Hermès, editor. Paris, France; 1995.

- [6] Melchior P, Lanusse P, Cois O, Dancla F, Oustaloup A. Crone toolbox for Matlab: fractional systems toolbox. In: Proceedings of the 41st IEEE CDC'02. Las Vegas, NV; 2002.
- [7] Podlubny I. Fractional differential equations, Mathematics in sciences and engineering no. 198, Academic Press; 1999.
- [8] Cois O, Oustaloup A, Battaglia E, Battaglia J-L. Non integer model from modal decomposition for time domain system identification. In: Proceedings of the 12th IFAC SYSID'2000, Santa-Barbara, CA; 2000.
- [9] Aoun M, Malti R, Levron F, Oustaloup A. Orthonormal basis functions for modeling continuous-time fractional systems. In: Proceedings of the 13th IFAC SYSID'03, Rotterdam, The Netherlands; 2003.
- [10] Montseny G. Diffusive representation of pseudo-differential time-operators. In: ESAIM proceedings, vol. 5, Paris, France; 1998.
- [11] Sathyanarayana S, Venugopalan S, Gopikanth ML. Impedance parameters and the state-of-charge I: Ni–Cd battery. J Appl Electrochem 1979;9:125–39.
- [12] Oldham KB, Spanier J. The fractional calculus. New York and London: Academic Press; 1974.
- [13] Miller KS, Ross B. An introduction to the fractional calculus and fractional differential equations. A Wiley-Interscience Publication; 1993.
- [14] Samko SG, Kilbas AA, Marichev OI. Fractional integrals and derivatives. Gordon and Breach Science Publishers; 1993.
- [15] Battaglia JL, Cois O, Puigsegur L, Oustaloup A. Solving an inverse heat conduction problem using a non-integer identified model. Int J Heat Mass Transfer 2001;44.
- [16] Marquardt DW. An algorithm for least squares estimation of non-linear parameters. J Soc Industr Appl Math 1963.
- [17] Söderstrom T, Stoica P. System identification. London: Prentice Hall International; 1989.
- [18] Ljung L. System identification: theory for the user. Prentice Hall; 1987.
- [19] Sabatier J, Aoun M, Oustaloup A, Grégoire G, Ragot F, Roy P. Fractional system identification for lead-acid battery state of charge estimation. Signal Process 2006;86:2645–57.
- [20] Hilfer R, editor. Applications of fractional calculus in physics. World Scientific; 2000.
- [21] Magin RL. Fractional calculus in bioengineering. Begell House, Connecticut; 2006.
- [22] Sabatier J, Agrawal O, Tenreiro Machado J. Advances in fractional calculus: theoretical developments and application in physics and engineering. Springer; 2007.
- [23] Nigmatullin RR, LeMehaute A. Is there a physical/geometrical meaning of the fractional integral with complex exponent? J Non-Cryst Solids 2005;351:2888–99.
- [24] Scalas E. Applications of continuous-time random walks in finance and economics. Physica A 2006;362(2):225–39.